

# STRESS INTENSITY FACTOR OF AN ARBITRARILY LOCATED CIRCUMFERENTIAL CRACK IN A THIN-WALLED CYLINDER WITH AXISYMMETRICALLY LOADED ENDS

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## ABSTRACT

Simplified method to approximately calculate the stress intensity factor of a circumferential crack in a thin-walled cylinder with ends subjected to axisymmetric radial and bending loads was developed, based on the theory of cylindrical shell and method similar to Rice and Levy's line spring method. The effects of cylinder length and crack location on the value can be evaluated by the method. The numerical results for the problem with the ends subjected to a pair of axisymmetric bending loads showed the necessity to consider the effects of cylinder length and crack location on the stress intensity factor for the problem. (100 words)

KEY WORDS : Fracture Mechanics, Stress Intensity Factor, Thin-Walled Cylinder, Axisymmetric Loads, Circumferential Crack, Compliance, Finite Length, Crack Location.

## 1. INTRODUCTION

The stress intensity factor (SIF) of a circumferential crack in a cylinder is one of the fundamental quantities to evaluate the reliability of a cracked pressure vessel. The SIF for this crack configuration under various loading conditions have been obtained by researchers and the SIF under uniform and linear stress distributions on the crack surface for a long cylinder have been summarized in handbooks [ 1 ].

Furthermore, as to the SIFs for a more general loading condition on the crack surface, there are literatures for an arbitrarily distributed stress on the crack surface [ 2 ], [ 3 ], in which the SIFs are given through the weight function for a long thin-walled cylinder derived by Labbens et al. [ 2 ] or the analytical method for an infinitely long thick-walled cylinder by Nied and Erdogan [ 3 ]. Labbens et al. [ 2 ] pointed out, in their paper, that the Mode I SIF of the crack is affected strongly by cylinder length and the assumption of long cylinder is satisfied with length  $H \geq 5/\beta$  ( $\beta$  is a quantity which is used in replacing cylindrical shell by a beam on an elastic foundation, and its definition is shown in chapter 2). So, if we intend to calculate the SIF of the crack in a short cylinder with axisymmetrically loaded ends, which is the case for a short-length internally pressurized cylindrical vessel with hemispherical ends, the information in these handbooks and literatures are not enough, because the SIF for this case may also be affected strongly by its length. In addition, also the location of the circumferential crack may affect the SIF, if the cylinder length has an effect on the value. Numerical methods such as FEM can be a candidate to calculate the SIF for such cases, but we felt an alternative method is necessary in systematically examining the effect of structural parameters, such as cylinder length, on the SIF.

In this paper, we developed a simple method to evaluate the SIF of a circumferential crack in a thin-walled cylinder with the ends subjected to axisymmetric bending and radial loads (Figure 1), based on the method similar to so-called Rice and Levy's line-spring method [ 4 ] and theory of cylindrical shell [ 5 ]. By applying the method developed, the effects that cylinder length  $H$  and crack location  $h_1/H$  have on the SIF can be evaluated systematically, with practically sufficient accuracy. Considering the fact that the actual cracked cylindrical vessels have finite length with a crack not necessarily located at the middle point of the cylinder length, this method will be useful in the field of design and inspection.

In the following, the ideas and basic way of thinking applied in deriving the simplified method are described first. After concrete formulation for a general case, the validity of the method is illustrated by comparing the SIF by the method with those by FEM for a problem under a pair of axisymmetric bending loads on both ends. From the numerical examples, the SIF for the problem

increased when the cylinder length decreased and when the crack is located near the cylinder end.

## 2. BASIC WAY OF THINKING

The way of thinking of the method derived in this paper will be introduced here. It is composed of three fundamental ideas and the last two are similar to the ideas of Rice and Levy's line spring method [ 4 ], although they were proposed independently by Okamura et al. [ 6 ], [ 7 ], [ 8 ] and Rice and Levy [ 4 ]. Their standings will be examined in the DISCUSSION. The concrete formulation of our method will be made in the following chapter.

### 2. 1 Replacement of an axisymmetric deformation problem of a cylinder by that of a beam on an elastic foundation

The fact "problems of axisymmetric deformation of cylindrical shells can be treated as those of lateral deformation of beams on elastic foundations" was focused at the beginning. That is, the deformation of a cylindrical shell under axisymmetric radial loads  $P_1$  and  $P_2$  and bending loads  $M_1$  and  $M_2$  on the edges (Figure 2, (a)) can be obtained as the deformation of a beam on an elastic foundation under the loads  $P_1$ ,  $P_2$ ,  $M_1$  and  $M_2$  on the ends (Figure 2, (b)), based on the analogy of their governing equations [ 5 ]. Here,  $P_1$ ,  $P_2$ ,  $M_1$  and  $M_2$  are defined as values per unit length in the circumferential direction for cylinders, and as values per unit thickness for the beams, respectively. Moreover, formally regarding  $D = EW^3/12(1-\nu^2)$  as the flexural rigidity of the prismatic beam, the value of  $k$  given by the following equation is employed as the spring constant of the elastic foundation.

$$k = 4\beta^4 D ; \beta^4 = \frac{EW}{4R_m^2 D} \quad (1)$$

where,  $R_m$  : mean radius,  $W$ : thickness,  $E$  : Young's Modulus,  $\nu$  : Poisson's Ratio.  $P_1$  and  $P_2$  were supposed to be loaded in the direction coinciding with one of the principal axes of the prismatic bar. In the same manner, the direction of the vectors  $M_1$  and  $M_2$  was assumed to coincide with the other principal axis. Note that  $1/\beta$  has a dimension of length.

### 2.2 Replacement of the cracked section

The knowledge that “the deformation of a cracked beam under lateral and bending loads, in which bending moment distributes in the longitudinal direction (Figure 3(a)), can be approximately evaluated by modeling the beam as two beams connected with a rotary spring of appropriate compliance (Figure 3(b))” [ 6 ], [ 7 ], [ 8 ] was focused next. Note that these beams do not necessarily have to be supported by an elastic foundation in general.

The above can be explained in the following way. Let us consider the deformation of a single edge cracked beam under pure bending (Figure 4). The thickness of the beam is unity. The inclination angle at the cross section, which is located  $l/2$  away from the crack,  $\theta$  can be formulated by moment  $M$  and compliance  $\lambda$  as,

$$\theta = \lambda \cdot M = (\lambda_0 + \Delta\lambda) \cdot M = \theta_0 + \Delta\theta \quad (2)$$

$$\theta_0 = \lambda_0 \cdot M, \quad \Delta\theta = \Delta\lambda \cdot M \quad (3)$$

where suffix 0 and symbol  $\Delta$  indicate the value without crack and the increment with crack, respectively.

Let us think about  $\Delta\theta$  at a cross section far away from the crack. As the part far away from the crack rotates like a rigid body, it can be said that  $\Delta\theta$  is almost caused by the increase in rotation of the section near the crack. When we take into account the fact  $\theta_0 = 0$  at the cracked section ( $l = 0$ ), the effect on the deformation due to the presence of the crack can be approximately treated by replacing the cracked section with a rotary spring of which the compliance is given by the increment of compliance due to the presence of the crack,  $\Delta\lambda$ , of an infinitely long beam under pure bending. Note that  $\Delta\lambda$  is affected by  $\xi = a/W$  and  $W$ , but not by  $l$ . By acceptance of these approximations, the presence of a single edge crack in a beam under arbitrarily distributed bending moment can also be treated as a rotary spring with the compliance  $\Delta\lambda$ , as far as the moment does not change its sign across the cracked section and the difference of moment across the cracked section is small [ 6 ], [ 7 ], [ 8 ].

### 2.3 The procedure to evaluate the required SIF

The SIF of a single edge cracked beam under arbitrarily distributed bending moment can be treated in a way similar to its deformation, which was described just before. That is, the moment on the spring

$M_C$  can be derived, once the cracked section of a beam is replaced by a spring (Figure 3). And almost the same moment as  $M_C$  is supposed to work on the section not so far from the crack (with small  $l/2$  in Figure 4), as far as the replacement by a spring above can be accepted. So “the SIF of a single edge cracked strip under distributed moment (Figure 3(a)) can be evaluated approximately as the SIF of a single edge cracked strip under pure bending moment  $M_C$ .”

The goal of this paper is to develop a simplified method to evaluate the SIF of a circumferential crack in a cylinder subjected to axisymmetric lateral and bending loads, as shown in Figure 1. This goal can be reached now by combining the three ideas that were discussed. That is, replace the cylinder with a beam on an elastic foundation (Figure 2), derive the moment  $M_C$  of the cracked section by replacing this section with a spring (Figure 3), and finally derive the SIF as the SIF of a single edge cracked strip under pure bending moment  $M_C$ .

### 3. FORMULATION

In this chapter, the problem of a circumferential crack in a thin-walled cylinder with the ends subjected to axisymmetric radial and bending loads shown in Figure 1 will be formulated as a problem of two beams on an elastic foundation connected with a spring shown in Figure 3 (b). Then a method to evaluate the SIF in a simplified way will be derived, according to the way of thinking in the previous section.

#### 3.1 Deformation of a beam on an elastic foundation under lateral and bending loads

As a first step, deformation of a beam on an elastic foundation of length  $h$  under lateral load  $P_l$  and bending load  $M_l$  on its left end will be discussed ( $P_2 = 0$  and  $M_2 = 0$  in Figure 5). Let the left and right ends of the beam be named points A and B, respectively. Here we focus our attention on point X, whose distances to points A and B are  $x$  and  $x'$ , respectively ( $x + x' = h$ ). The deflection  $y$  and inclination angle  $\theta$  of point X can be expressed, by introducing compliance  $\lambda$  and compliance matrix  $\Lambda(x, x')$ , as

$$\begin{bmatrix} y(x, x') \\ \theta(x, x') \end{bmatrix} = \begin{bmatrix} \lambda_{yP}(x, x') & \lambda_{yM}(x, x') \\ \lambda_{\theta P}(x, x') & \lambda_{\theta M}(x, x') \end{bmatrix} \cdot \begin{bmatrix} P_l \\ M_l \end{bmatrix} \equiv \Lambda(x, x') \cdot \begin{bmatrix} P_l \\ M_l \end{bmatrix} \quad (4)$$

where  $\lambda_{yP}$ , for instance, means the compliance that relates  $P$  to  $y$  caused by  $P$ .

Each component of the matrix is as shown in Eqs. ( 5 ) to ( 8 ). These were originally derived by Hetényi[ 9 ], and rewritten here in a format of a cylindrical shell.

$$\lambda_{yP}(x, x') = \frac{1}{2\beta^3 D} \times \frac{\sinh \beta h \cos \beta x \cosh \beta x' - \sin \beta h \cosh \beta x \cos \beta x'}{\sinh^2 \beta h - \sin^2 \beta h} \quad (5)$$

$$\lambda_{\theta P}(x, x') = -\frac{1}{2\beta^2 D} \times \frac{1}{\sinh^2 \beta h - \sin^2 \beta h} \times [\sinh \beta h (\sin \beta x \cosh \beta x' + \cos \beta x \sinh \beta x') + \sin \beta h (\sinh \beta x \cos \beta x' + \cosh \beta x \sin \beta x')] \quad (6)$$

$$\lambda_{yM}(x, x') = \frac{1}{2\beta^2 D} \times \frac{1}{\sinh^2 \beta h - \sin^2 \beta h} \times [\sinh \beta h (\sin \beta x \cosh \beta x' - \cos \beta x \sinh \beta x') + \sin \beta h (\sinh \beta x \cos \beta x' - \cosh \beta x \sin \beta x')] \quad (7)$$

$$\lambda_{\theta M}(x, x') = \frac{1}{\beta D} \times \frac{\sinh \beta h \cos \beta x \cosh \beta x' + \sin \beta h \cosh \beta x \cos \beta x'}{\sinh^2 \beta h - \sin^2 \beta h} \quad (8)$$

By virtue of superposition principle and the symmetry of the beam and sustaining conditions, the deflection and inclination angle at point X under the loading condition shown in Figure 5 can be expressed in the following way.

$$\begin{bmatrix} y(x, x') \\ \theta(x, x') \end{bmatrix} = \Lambda(x, x') \cdot \begin{bmatrix} P_1 \\ M_1 \end{bmatrix} + \Lambda_*(x, x') \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \quad (9)$$

where the matrix  $\Lambda_*(x, x')$  is defined as follows.

$$\Lambda_*(x, x') \equiv \begin{bmatrix} \lambda_{yP}^*(x, x') & \lambda_{yM}^*(x, x') \\ \lambda_{\theta P}^*(x, x') & \lambda_{\theta M}^*(x, x') \end{bmatrix} = \begin{bmatrix} \lambda_{yP}(x', x) & \lambda_{yM}(x', x) \\ -\lambda_{\theta P}(x', x) & -\lambda_{\theta M}(x', x) \end{bmatrix} \quad (10)$$

Next, we will forward to the problem of “two beams on an elastic foundation connected by a spring with an appropriate compliance in Figure 3(b).” In this case, although the deflection should be continuous across the spring position, its derivative, that is, the inclination angle is discontinuous there because of the existence of rotary spring. Here, let us denote the shearing force and bending moment at the spring position by  $F_C$  and  $M_C$ , respectively, as shown in Figure 6 and apply Eq. ( 9 ) to two beams in the figure. The deflection and inclination angle at point C, considering  $h = h_1$ ,  $x = h_1$  and  $x' = 0$  for beam AC and  $h = h_2$ ,  $x = 0$  and  $x' = h_2$  for beam CB, are given by

$$\begin{bmatrix} y_C \\ \theta_{C1} \end{bmatrix} = \Lambda(h_1, 0) \cdot \begin{bmatrix} P_1 \\ M_1 \end{bmatrix} + \Lambda_*(h_1, 0) \cdot \begin{bmatrix} F_C \\ M_C \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} y_C \\ \theta_{C2} \end{bmatrix} = \Lambda(0, h_2) \cdot \begin{bmatrix} -F_C \\ M_C \end{bmatrix} + \Lambda_*(0, h_2) \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \quad (12)$$

by applying Eq. ( 9 ) to beams AC and CB, respectively. Here, as there exists the discrepancy between the inclination angles at point C for two beams,  $\theta_{C1}$  for beam AC is discriminated from  $\theta_{C2}$  for beam CB. The discrepancy, by introducing the compliance  $\Delta\lambda$  of the spring, is related to bending moment  $M_C$  as shown in the following equation.

$$\theta_{C2} - \theta_{C1} = -2\Delta\lambda \cdot M_C \quad (13)$$

These equations from Eq. ( 11 ) to Eq. ( 13 ) set up a set of simultaneous equations which have five equations and unknown variables  $F_C$ ,  $M_C$ ,  $y_C$ ,  $\theta_{C1}$  and  $\theta_{C2}$ . Therefore, these variables are obtained and given by

$$\mathbf{F}_{Cg} = \mathbf{C}_g^{-1} \times \mathbf{B}_g \times \mathbf{P}_{ABg} \quad (14)$$

The matrixes in Eq. ( 14 ) are as follows.

$$\mathbf{F}_{Cg} = [F_C \quad M_C \quad y_C \quad \theta_{C1} \quad \theta_{C2}]^t \quad (15)$$

$$\mathbf{C}_g = \begin{bmatrix} \lambda_{yP}^*(h_1, 0) & \lambda_{yM}^*(h_1, 0) & -1 & 0 & 0 \\ \lambda_{\theta P}^*(h_1, 0) & \lambda_{\theta M}^*(h_1, 0) & 0 & -1 & 0 \\ -\lambda_{yP}(0, h_2) & \lambda_{yM}(0, h_2) & -1 & 0 & 0 \\ -\lambda_{\theta P}(0, h_2) & \lambda_{\theta M}(0, h_2) & 0 & 0 & -1 \\ 0 & -2\Delta\lambda & 0 & 1 & -1 \end{bmatrix} \quad (16)$$

$$\mathbf{B}_g = \begin{bmatrix} -\lambda_{yP}(h_1, 0) & -\lambda_{yM}(h_1, 0) & 0 & 0 \\ -\lambda_{\theta P}(h_1, 0) & -\lambda_{\theta M}(h_1, 0) & 0 & 0 \\ 0 & 0 & -\lambda_{yP}^*(0, h_2) & -\lambda_{yM}^*(0, h_2) \\ 0 & 0 & -\lambda_{\theta P}^*(0, h_2) & -\lambda_{\theta M}^*(0, h_2) \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

$$\mathbf{P}_{ABg} = [P_1 \quad M_1 \quad P_2 \quad M_2]^t \quad (18)$$

### 3. 2 The SIF of a cracked beam on an elastic foundation

Once the moment on spring  $M_C$  is obtained from Eq. ( 14 ), the SIF of the problem can be calculated as the SIF of a single edge cracked strip under pure bending  $M_C$ ,

$$K_M = \frac{M_C}{Z} \sqrt{\pi a} \cdot F_M(\xi) \quad (19)$$

where  $Z = W^2/6$  and  $F_M(\xi)$  is a correction factor for finite width under pure bending.

As the compliance of the beams defined in Eqs. ( 5 ) to ( 8 ) were formulated in a form applicable to a cylindrical shell, we have reached our goal. That is, the method described here to evaluate the SIF can be directly used to evaluate the SIF of a circumferentially cracked thin-walled cylinder with ends subjected to axisymmetric loads shown in Figure 1.

### 3. 3 The deflection and the inclination angle at the end of a cracked beam on an elastic foundation

Here, the deflections and the inclination angles at the end points A and B of the cracked beam on an elastic foundation will be derived (Figure 6) . We will start on the basis that the loads on the point C,  $F_C$  and  $M_C$ , are known, by solving Eq. ( 14 ). By applying Eq. ( 9 ) on beams AC and CB, the deformation at the ends are directly derived.

$$\begin{bmatrix} y_A \\ \theta_A \end{bmatrix} = \Lambda(0, h_1) \cdot \begin{bmatrix} P_1 \\ M_1 \end{bmatrix} + \Lambda_*(0, h_1) \cdot \begin{bmatrix} F_C \\ M_C \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} y_B \\ \theta_B \end{bmatrix} = \Lambda(h_2, 0) \cdot \begin{bmatrix} -F_C \\ M_C \end{bmatrix} + \Lambda_*(h_2, 0) \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \quad (21)$$

Suffix A and B show the location.

## 4. NUMERICAL ILLUSTRATION

To illustrate the validity of Eq. ( 19 ), the SIF calculated by this equation  $K_M$  was compared with the SIF  $K_{FEM}$  by FEM for the case of equal axisymmetric bending loads at both ends of the cylinder. In this case, the required  $M_C$  in Eq. ( 19 ) can be calculated by adding the conditions  $P_1 = P_2 = 0$ ,  $M_1 = M_2 = M$  to the Eqs. ( 14 ) to ( 18 ), and by solving Eq. ( 14 ).

The case that was investigated is as follows. The cylinder has mean radius  $R_m = 105$  mm and thickness  $W = 10$  mm for all cases, and, as to material constants, Young's Modulus  $E = 206$  GPa, Poisson's



Ratio  $\nu = 0.3$  are commonly used. Two cases were investigated for the total length of the cylinder, that is  $H = 40$  and  $100$  mm. For each  $H$ , crack location was varied for  $h_1/H = 0.5, 0.625$  and  $0.75$ . The calculated results are normalized by  $K_{Mbeam}$ , which is the SIF of a single edge cracked strip under pure bending moment  $M$ , and are compared in Figure 7.

From this figure, the following can be read.

1. These two solutions show good agreement in a practical sense even when  $H/W = 4$  and  $h_1/H = 0.75$ , therefore,  $h_2 = W$ .
2. The SIF in interest,  $K_M$  becomes small for long cylinders.
3.  $K_M$  becomes large when the crack is located near the edge of a cylinder.
4. It is necessary to consider the effects of the cylinder length and the crack location, in evaluating the SIF of a circumferentially cracked cylinder under axisymmetric bending.

The correction factor for finite width  $F_M$  [ 1 ] and spring compliance  $\Delta\lambda$  [ 10 ], which were used in the numerical examples are as follows.

$$F_M(\xi) = \sqrt{\frac{2}{\pi\xi} \tan \frac{\pi\xi}{2}} \cdot \frac{0.923 + 0.199[1 - \sin(\pi\xi/2)]^4}{\cos(\pi\xi/2)} \quad (22)$$

$$\Delta\lambda(\xi) = \frac{\pi(1.1215)^2}{2E} \cdot \frac{\xi^2}{(1-\xi)^2(1+2\xi)^2} \times [1 + \xi(1-\xi)(0.44 + 0.25\xi)] \left(\frac{6}{W}\right)^2 \quad (23)$$

## 5. DISCUSSION

The idea of replacing the cracked cross section by an elastic spring, which is referred as “Rice and Levy’s line spring method” nowadays, was first proposed by Okamura et al. [ 6 ] and was developed independently by their group [ 7 ], [ 8 ] and Rice and Levy [ 4 ]. As Rice and Levy [ 4 ] pointed out, Okamura et al. [ 6 ] did not consider rotation due to axial load, but was thought enough for the problem considered. Recently, Valiente et al. [ 6 ] formulated the statically indeterminate cracked beam problem (without elastic foundation) including the shear deformation of the spring, while this effect was not considered in this paper. However, the numerical results indicate the validity of neglecting these terms in our formulation for the problem treated in this paper.

Furthermore, as the method developed here is based on the beam theory, accurate solution is expected in general for long beams. However, it seems that the method can be applied even to the case where  $h_1$  or  $h_2$  becomes comparable to  $W$  and the beam theory no longer holds.

As to the characteristics of the SIF for the circumferential crack in a thin-walled cylinder with ends subjected to bending load pair, the cylinder length and the crack location affected the SIF strongly. This indicates the necessity to consider the effects of cylinder length and crack location on the SIF appropriately, while the well-known handbooks do not point out this fact explicitly.

## 6. CONCLUSIONS

In this paper, a simplified method to evaluate the SIF of a circumferential crack in a thin-walled cylinder with ends subjected to axisymmetric radial and bending loads was derived theoretically. The effects that the cylinder length and the crack location have on the SIF, can be evaluated by the method. The validity of the method was illustrated by comparing the solutions with the numerical ones, for a problem under a pair of axisymmetric bending loads on both ends. These two solutions showed good agreement in a practical sense. In addition, the results showed that the SIF increased when the cylinder length decreased and when the crack is located near the cylinder edge. These warn us to take into account the effects of the cylinder length and the crack location appropriately, in evaluating the SIF of a circumferentially cracked thin-walled cylinder with ends subjected to axisymmetric bending loads.

## REFERENCES

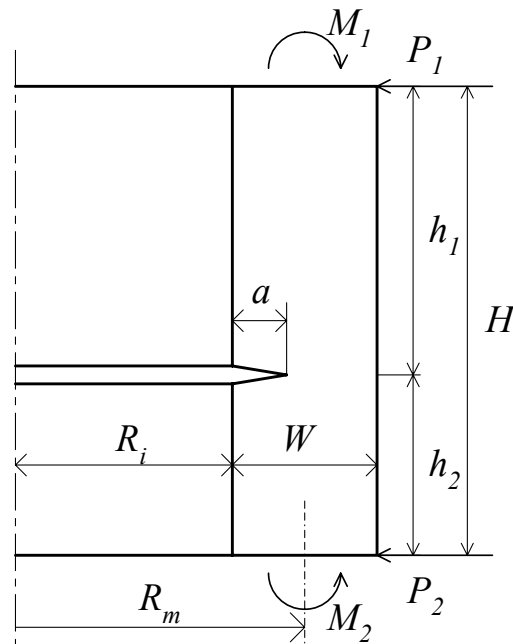
- [ 1 ] e. g., Tada, H., Paris, P. C. and Irwin, G.R., *The Stress Analysis of Cracks Handbook*, Del Research Corporation, Hellertown, 1973.
- [ 2 ] Labbens, R., Pellissier-Tanon, A. and Helliot, J., Practical method for calculating stress-intensity factors through weight functions. *ASTM STP*. 1976, **590**, 368-384.
- [ 3 ] Nied, H. F. and Erdogan, F., The elasticity problem for a thick-walled cylinder containing a circumferential crack. *Int J Fracture*. 1983, **22**, 277-301.
- [ 4 ] Rice, J. R. and Levy, N., The part-through surface crack in an elastic plate. *J Appl Mech. Trans*

*ASME*. 1972, **39**, 185-194.

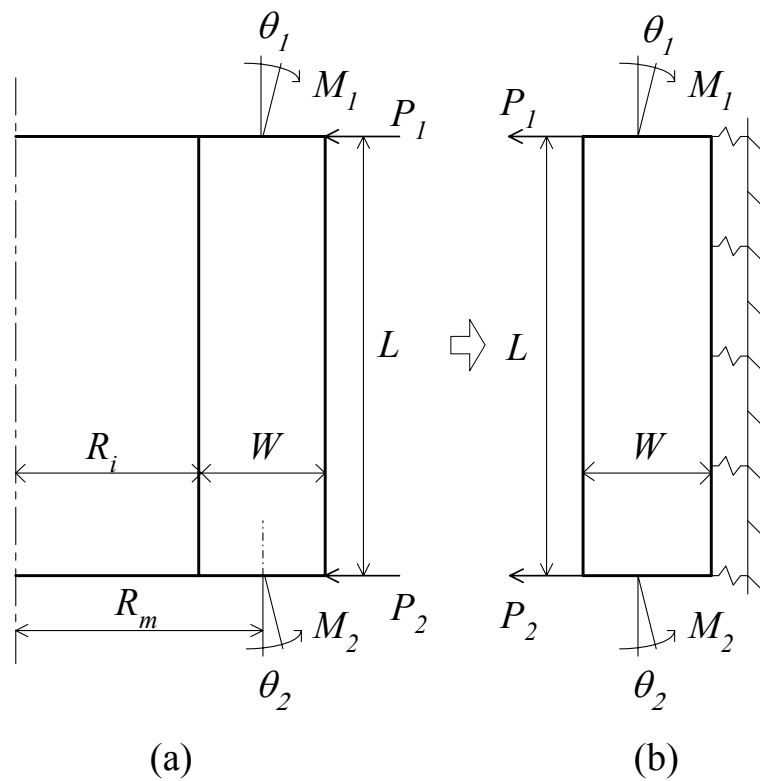
- [ 5 ] e. g., Timoshenko, S. P., *Strength of Materials*, Part II, 2nd Printing, D Van Nostrand Company, 1934, Chap I.
- [ 6 ] Okamura, H., Liu, H. W., Chu, C-S. and Liebowitz, H., A cracked column under compression. *Engng Fracture Mech.* 1969, **1**, 547-564.
- [ 7 ] Okamura, H., Watanabe, K. and Takano, T., Application of the compliance concept in fracture mechanics. *ASTM STP*. 1973, **536** , 423-438.
- [ 8 ] Okamura, H., Watanabe, K. and Naito, Y., Some crack problems in structural reliability analysis in: *Reliability Approach in Structural Engineering* (Edited by M. Freudenthal, M. Shinozuka, I. Konishi and T. Kanazawa), Maruzen, Tokyo, 1975, 243-257.
- [ 9 ] Hetényi, M., *Beams on Elastic Foundation*, 2nd Edition, The University of Michigan Press, Ann Arbor, 1971.
- [ 10 ] Takahashi, J., *Research on Linear and Non-Linear Fracture Mechanics based on Energy Principle* (in Japanese), Ph. D. Dissertation, Univ. of Tokyo, 1991.
- [ 11 ] Valiente, A., Elices, M. and Ustáriz, F., Stress intensity factors in statically indeterminate cracked beams in: *Structural Integrity: Experiments - Models - Applications* (Edited by K. H. Schwalbe and C. Berger), 1994, 781-786.

## LIST OF FIGURES

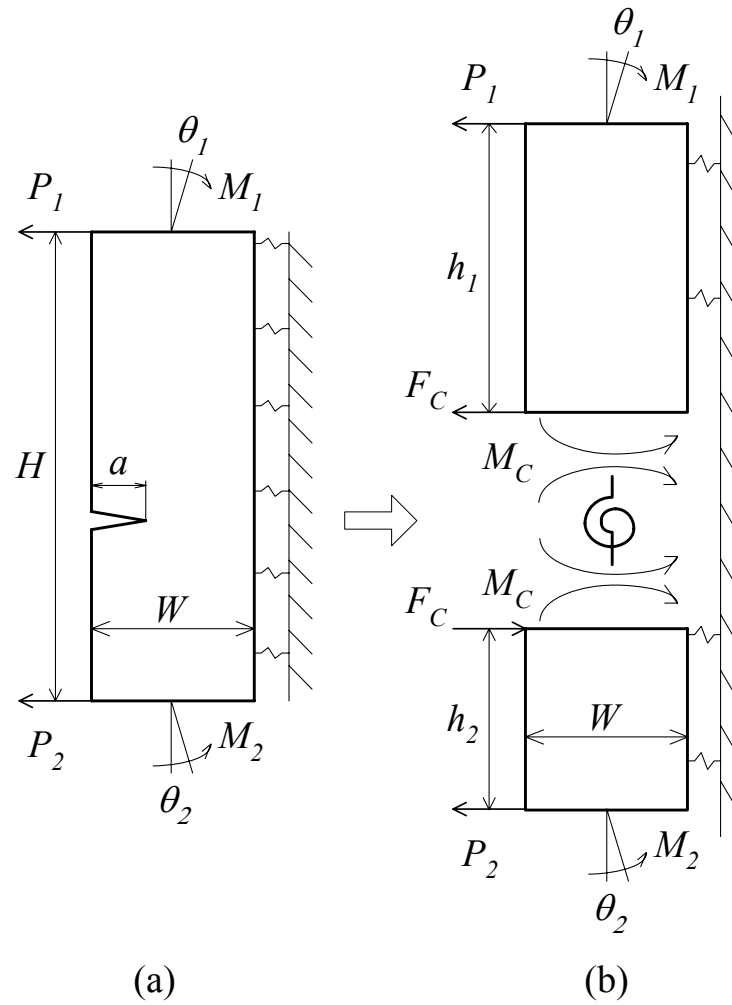
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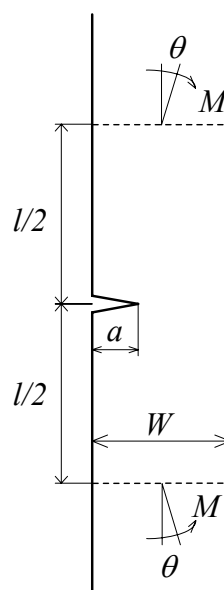
**Figure 1** A circumferentially cracked cylinder under axisymmetric loads



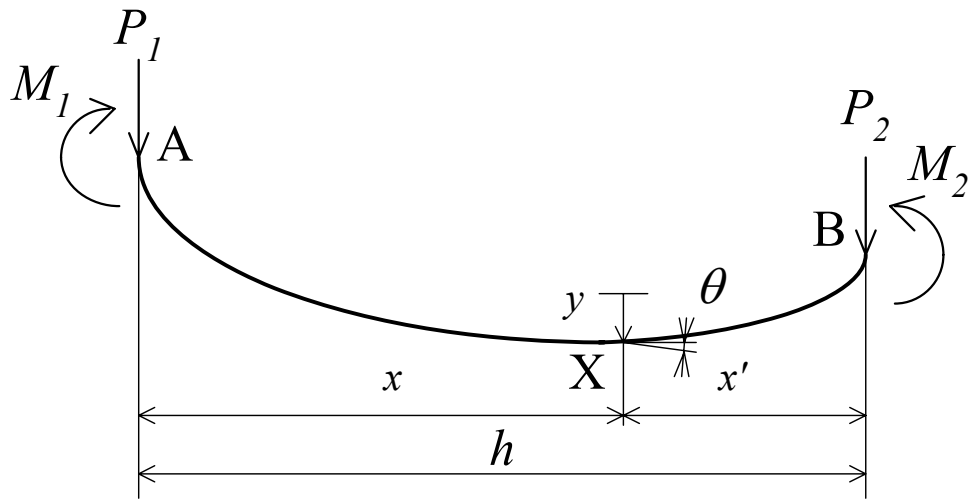
**Figure 2** Replacement of axisymmetric deformation problem of a thin-walled cylinder by that of a beam on an elastic foundation



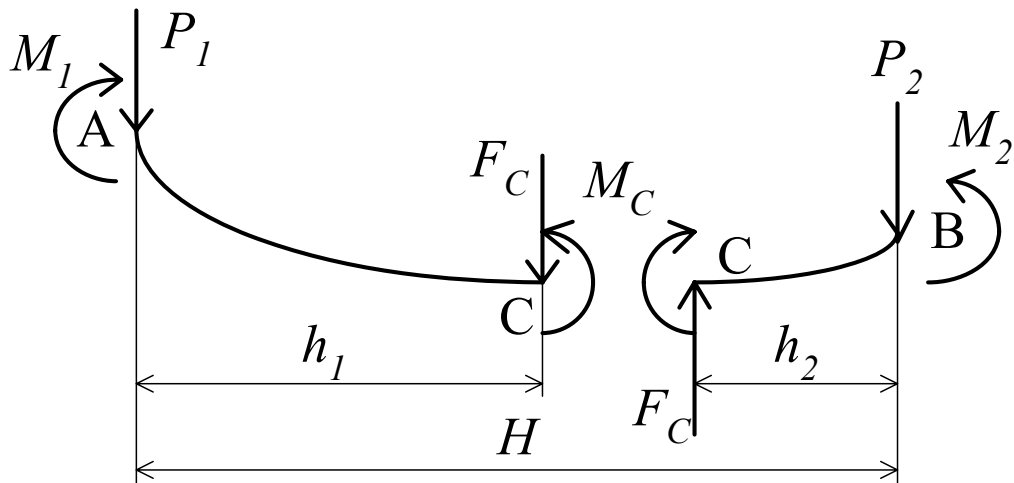
**Figure 3 Replacement of the cracked section by a spring**



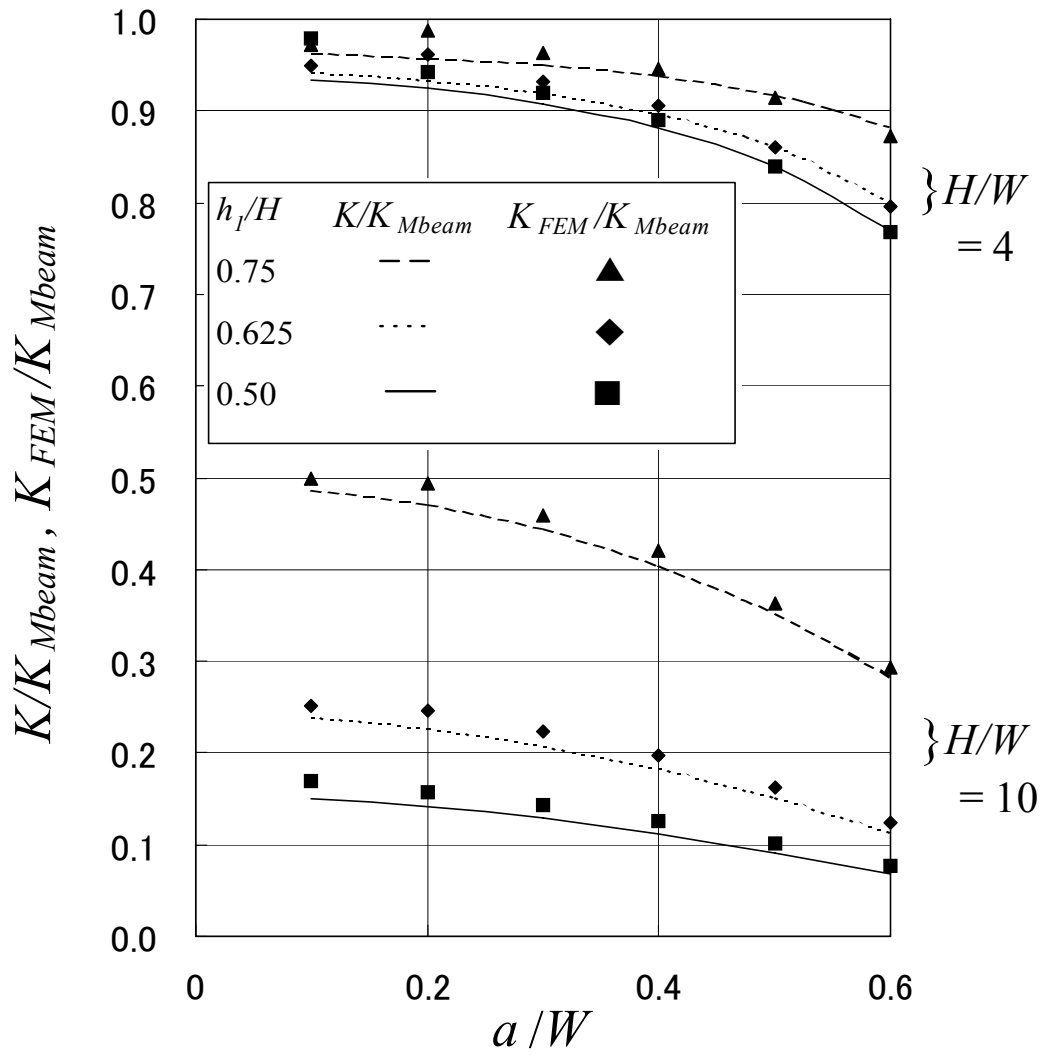
**Figure 4 A cracked beam under pure bending**



**Figure 5** A beam on an elastic foundation under lateral and bending loads on both ends



**Figure 6** Shearing force and bending moment at the spring position



**Figure 7** Comparison of the SIF by the simplified method with that from FEM